Reg. No. :

Question Paper Code : 50776

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2017
Second Semester
Civil Engineering
MA 6251 – MATHEMATICS – II
(Common to Mechanical Engineering (Sandwich)/Aeronautical Engineering/ Agriculture Engineering/Automobile Engineering/Biomedical Engineering/ Computer Science and Engineering/Electrical and Electronics Engineering/ Electronics and Communication Engineering/Electronics and Instrumentation Engineering/Environmental Engineering/Geoinformatics Engineering/Industrial Engineering and Management/Instrumentation and Control Engineering/ Manufacturing Engineering/Materials Science and Engineering/Mechanical Engineering/Mechanical and Automation Engineering/Mechatronics Engineering/ Medical Electronics Engineering/Petrochemical Engineering/Production Engineering/Robotics and Automation Engineering/Biotechnology/Chemical Engineering/Chemical and Electrochemical Engineering/Fashion Technology/Food Technology/Handloom & Textile Technology/Information Technology/ Petrochemical Technology/Petroleum Engineering/Pharmaceutical Technology/ Plastic Technology/Polymer Technology/Textile Chemistry/Textile Technology)
(Regulations 2013)

Time : Three Hours | Maximum : 100 Marks

Answer ALL questions.

PART – A (10×2=20 Marks)

1. If $\vec{F}=(x+3y)\vec{i}+(y-2z)\vec{j}+(x+2kz)\vec{k}$ has divergence zero, find the unknown value of k. (2)

2. Evaluate the integral $\int_C \vec{F}.d\vec{r}$ if $\vec{F}=xy^2\vec{i}+(x^2+y^2)\vec{j}$ and C is the curve given by $y=x^2-4$ from (2, 0) to (4, 12). (2)

3. Evaluate $\frac{1}{D^2-6D+9}\{3\log 2\}$ (2)

4. Solve $x^2y'' + 4xy' + 2y = 0$. (2)

5. Find $L[f(t)]$ if $f(t)=\begin{cases} e^{-t}, 0<t<4 \\ 0, \ t>4 \end{cases}$ (2)

6. Find $f(\infty)$, if $L[f(t)]=\frac{1}{s(s+\alpha)}$. (2)

7. Examine whether $y + e^x \cos y$ is harmonic. (2)

8. Find the image of the line $x = 1$ under the transformation $w = z^2$. (2)

9. Expand $\frac{z-1}{z+1}$ about $z = 1$. (2)

10. Find the singular points of $f(z)=\frac{\sin z}{z}$ (2)

PART – B (5×16=80 Marks)

11. a) i) Prove that $\vec{F}=(x^2-y^2+x)\vec{i}-(2xy+y)\vec{j}$ is a conservative field and find the scalar potential of $\vec{F}$. (8)

ii) Apply Green's theorem to evaluate $\int_C (xy-x^2)dx+x^2y\,dy$ along the closed curve C formed by $y = 0$, $x = 1$ and $y = x$. (8)

(OR)

b) i) Evaluate $\iint_S \vec{F}.\hat{n}\,dS$ where $\vec{F}=z\vec{i}+x\vec{j}-3y^2z\vec{k}$ and S is the surface of the cylinder $x^2 + y^2 = 16$ included in the first octant between $z = 0$ and $z = 5$. (8)

ii) Evaluate $\iint \vec{F}.\hat{n}\,dS$ using Gauss divergence theorem for $\vec{F}=x^2\vec{i}+y^2\vec{j}+z^2\vec{k}$ taken over the cube bounded by the planes $x = 0$, $y = 0$, $z = 0$, $x = 1$, $y = 1$, $z = 1$. (8)

12. a) i) Solve $(D^3 - 5D^2 + 7D - 3)y = e^{2x}\cosh x$ where $D = \frac{d}{dx}$ (8)

ii) Solve $(D + 2)\,x + 3y = 0$, $(D + 2)\,y + 3x = 2e^{2t}$ where $D = \frac{d}{dt}$. (8)

(OR)

b) i) Solve $(x + 1)^2 D^2 + (x + 1) D + y = \sin 2(\log(x + 1))$ where $D = \frac{d}{dx}$. (8)

ii) Solve by method of variation of parameters $y'' - 6y' + 9y = \frac{e^{3x}}{x^2}$ where $D = \frac{d}{dx}$. (8)

13. a) i) Evaluate

i) $\int_0^\infty te^{-2t} t \sin 3t\,dt$ using Laplace transform and

ii) $L^{-1}\left[\cot^{-1}\left(\frac{2}{s+1}\right)\right]$ (8)

ii) Find $L^{-1}\left[\frac{1}{(s^2+a^2)^2}\right]$ using convolution theorem. (8)

(OR)

b) i) Find the Laplace transform of a square wave defined by

$$f(t) = \begin{cases} E, & 0 < t < \frac{a}{2}, \\ -E, & \frac{a}{2} < t < a, \end{cases} \text{ and } f(t) = f(t+a)$$ (8)

ii) Using Laplace transform, solve $y'' + y' = t^2 + 2t$ when $y(0) = 4$, $y'(0) = -2$. (8)

14. a) i) Prove that $f(z) = z^n$ is analytic for all values of n and find its derivative. (8)

ii) Find the bilinear transformation which maps the points $z = 1, i, -1$ in to the points $w = i, 0, -i$. Hence find the image of $|z| < 1$. (8)

(OR)

b) i) Prove that the function $u = \log\sqrt{x^2+y^2}$ is harmonic and hence find its conjugate harmonic (8)

ii) Find the image of the circle $|z-2i|=2$ under the transformation $w=\frac{1}{z}$. (8)

15. a) i) Evaluate $\int_C \frac{\cos\pi z}{z^2-1}\,dz$ around a rectangle with vertices at $2\pm i, -2\pm i$. (8)

ii) Evaluate $\int_0^{2\pi} \frac{1}{2+\cos\theta}\,d\theta$ using contour integration. (8)

(OR)

b) i) Expand $\frac{1}{(z-1)(z-2)}$ in a Laurent series valid for

(i) $|z| < 1$, ii) $1 < |z| < 2$. (8)

ii) Use calculus of residues to find $\int_0^{\infty} \frac{1}{(x^2+a^2)(x^2+b^2)}\,dx$ where $a, b > 0$. (8)